

How to find an Inverse Function

Inverse Function – A function derived from an original function in which each input becomes an output, and each output becomes an input for the function.

Example 1: Inverse Relations/ordered pairs:

Find the inverse of the relation:

$\{(1,2), (2,4), (7, -2), (9, -3), (10,6)\}$

Solution:

Interchange each x and y value and rewrite the relation:

$\{(2, 1), (4, 2), (-2, 7), (-3, 9), (6, 10)\}$

Example 2: Inverse Linear Functions

Find the inverse of the following function:

$f(x) = 8x + 9$

Solution:

Step 1: Replace $f(x)$ with y :

$$y = 8x + 9$$

Step 2: Interchange x and y :

$$x = 8y + 9$$

Step 3: Solve for y (Subtract 9 from each side):

$$x - 9 = 8y + 9 - 9 \quad \rightarrow \quad x - 9 = 8y$$

Step 4: Divide by 8 on each side:

$$\frac{x-9}{8} = \frac{8y}{8} \quad \rightarrow \quad \frac{x-9}{8} = y$$

Step 4: Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \frac{x-9}{8}$$

Example 3:

Find the inverse of a rational function:

$$f(x) = \frac{2}{x+3}$$

Step 1: Replace $f(x)$ with y :

$$y = \frac{2}{x+3}$$

Step 2: Interchange x and y :

$$x = \frac{2}{y+3}$$

Step 3: Solve for y (Multiply by $y + 3$ on each side):

$$x(y+3) = \frac{2(y+3)}{(y+3)} \quad \rightarrow \quad x(y+3) = 2$$

Step 4: Divide by x on each side:

$$\frac{x}{x}(y+3) = \frac{2}{x} \quad \rightarrow \quad y+3 = \frac{2}{x}$$

Step 5: Subtract 3 on each side:

$$y+3 - 3 = \frac{2}{x} - 3 \quad \rightarrow \quad y = \frac{2}{x} - 3$$

Step 6: Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \frac{2}{x} - 3$$

Practice Problems

Find the inverse of the following functions:

1) $h(x) = x^2 - 4$

2) $f(x) = \frac{4x-1}{-2x+3}$

3) $g(x) = \sqrt{5x-6}$

Solutions:

1) $h^{-1}(x) = \sqrt{x+4}$

2) $f^{-1}(x) = \frac{3x+1}{2x+4}$

3) $g^{-1}(x) = \frac{y^2+6}{5}$