

Slope-Intercept Form and Point-Slope Form

Slope of the line	$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form	$y = mx + b$; where m is slope and b is the y-intercept
Point-Slope Form	$y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$
Slope of parallel lines	$m_1 = m_2$ (slopes are the same)
Slope of perpendicular lines	$m_1 = -\frac{1}{m_2}$ (slopes are opposite & reciprocal)
Equations of Horizontal and Vertical Lines	Horizontal: $y = b$ Vertical: $x = a$; where a is the x intercept

Example (1): Write the slope - intercept equation of a line which passes through (0,-7) whose slope is 2.

Solution: Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope (m) & y-intercept (b), and the problem provides both parts.

$$m = 2 \text{ and } b = -7 \text{ The equation of the line is } y = 2x - 7$$

Example (2): Write the slope-intercept equation of a line which passes through (0,4) and (3,-5).

Solution: Slope-intercept equation is $y = mx + b$ What we need to complete this equation are slope (m) & y-intercept (b), however, we only have y-intercept. To find the slope,

$$m = \frac{\text{rise}}{\text{run}} = \frac{-5 - 4}{3 - 0} = \frac{-9}{3} = -3$$

$$m = -3 \text{ and } b = 4 \text{ The equation of the line is } y = -3x + 4$$

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Example (3): Write the slope-intercept equation of a line which passes through $(-1, 4)$ whose slope is 5.

Solution: Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope (m) & y-intercept (b), however, we only have slope. Here there are two ways to find the equation of the line.

Method I We will substitute m and (x_1, y_1) into the equation $y = mx + b$.

$$m = 5 \text{ and } (x_1, y_1) = (-1, 4)$$

$$4 = 5(-1) + b$$

$$4 = -5 + b$$

$$9 = b$$

$$y = 5x + 9$$

Method II Since we are given slope m and an ordered pair (x_1, y_1) , we can use Point-slope form $y - y_1 = m(x - x_1)$ to find equation of the line.

$$m = 5 \text{ and } (x_1, y_1) = (-1, 4)$$

$$y - 4 = 5(x - (-1))$$

$$y - 4 = 5(x + 1)$$

$$y - 4 = 5x + 5$$

$$y = 5x + 5 + 4$$

$$y = 5x + 9$$

Example (4): Write the slope-intercept equation of a line which passes through $(1, 3)$ and $(-5, -1)$

Solution: Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope & y-intercept. However, we are given two ordered pairs (x_1, y_1) and (x_2, y_2) without slope or y-intercept. Therefore, we need to find the slope first. Then we can use the two methods discussed on Example (3) to find the equation of the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-1 - 3}{-5 - 1} = \frac{-4}{-6} = \frac{2}{3}$$

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Method I Now we have slope, we will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b .

$$\begin{aligned} y &= mx + b \\ 3 &= \frac{2}{3}(1) + b \\ 3 &= \frac{2}{3} + b \\ 3 - \frac{2}{3} &= b \\ \frac{9}{3} - \frac{2}{3} &= b \\ \frac{7}{3} &= b \\ y &= \frac{2}{3}x + \frac{7}{3} \end{aligned}$$

Method II We also can use Point-slope form to find the equation of the line.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= \frac{2}{3}(x - 1) \\ y - 3 &= \frac{2}{3}x - \frac{2}{3} \\ y &= \frac{2}{3}x - \frac{2}{3} + 3 \\ y &= \frac{2}{3}x + \frac{7}{3} \end{aligned}$$

Example (5): Write the equations of a parallel and perpendicular line to the equation: $y = 2x + 1$.

Parallel: Parallel lines have equal slopes, so any line with $m = 2$ is parallel.

Perpendicular: Perpendicular lines have a negative reciprocal slope. Any line with $m = -\frac{1}{2}$ will be perpendicular.

Example (6): Write the equations of a vertical line and a horizontal line passing through $(3,4)$.

Vertical: $x = 3$

Horizontal: $y = 4$

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Exercises:

1. Write the slope - intercept equation of a line which passes through (1,3) whose slope is 4.
2. Write the slope-intercept equation of a line which passes through (1,2) and (5,3).
3. Write the slope-intercept equation of a line which is parallel to $y = 4x + 1$, passing through (8,2).
4. Write the slope-intercept equation of a line which is perpendicular to $y = -x + 2$, passing through (-1,1).
5. Write an equation of a horizontal line which passes through (-2, -4).
6. Write an equation of a vertical line which passes through (-5,6).

Answers:

1. $y = 4x - 1$
2. $y = \frac{1}{4}x + \frac{7}{4}$
3. $y = 4x - 30$
4. $y = x + 2$
5. $y = -4$
6. $x = -5$