TSC | WILLIAM D. LAW, JR. LEARNING COMMONS

# Factoring Trinomials/Quadratics

Most trinomials that you will deal with in this class will come in the form of a quadratic. All quadratics are of the following form:

$$ax^2 + bx + c$$

However there are different methods to solve when the *a* term is and is not 1.

I. Quadratics with a leading term of 1

Quadratics with a leading term of 1 can be solved a number of ways. We will demonstrate using the box method and "guess and check".

**Example:**

$$x^2 + 5x + 6$$

First write out the factors of 6. (This is the list of numbers that multiply to 6)

$$6: 1,2,3,6$$

Then find the ones that could add to 6 (if 6 was negative, we would be looking for one positive and negative factor)

$$5 = 2 + 3 \; and \; 6 = 2 * 3$$

Now split your middle term $bx$ into two parts

$$x^2 + 2x + 3x + 6$$

Now use a box to separate your terms

| | |
|---|---|
| $ax^2$ | $+b_1x$ |
| $+b_2x$ | $+c$ |

| | |
|---|---|
| $x^2$ | $+2x$ |
| $+3x$ | $+6$ |

Find the GCF of each row and each column. (The GCF is the largest amount of constant and variable that you could take out of all terms being considered. i.e. $4x^2y^5$ and $6x^3y^3$have a GCF of $2x^2y^3$)

| GCF | $(\boldsymbol{x}$ | $+\mathbf{2})$ |
|---|---|---|
| $(\boldsymbol{x}$ | $x^2$ | $+2x$ |
| $+\mathbf{3})$ | $+3x$ | $+6$ |

The factored form of $x^2 + 5x + 6$ is $(x + 3)(x + 2)$

**Using guess and check:**

Guess and check is very similar.

First you consider that this is the factored form of any quadratic:

$$(x+m)(x+n)$$

Given that $m$ and $n$ are real numbers. So, we must consider that if we were to FOIL this and multiply these together, we would end up with:

$$x^2+mx+nx+mn$$

So from this, we can devise that we need $m+n=b$ and $m*n=c$. If $m$ and $n$ meet those conditions, then those are the factors.

**Example:**

$$x^2-3x-4$$

Consider our factors of -4 (Note: It is important to note that since the 4 is negative, we have to consider cases where one factor is positive and one is negative.)

$$-4:\ \pm 1, \pm 2, \pm 4$$

Given that our goal is $-3$, we must only consider pairs where the larger number is negative. This leaves us with $-2, +2$ and $-4, +1$ as our only options for $m, n$ respectively.

So now we can simply plug in the numbers and check.

$$-2+2=-3$$

$$0=-3 \text{ False}$$
$$-4+1=-3$$
$$-3=-3 \text{ True}$$

Therefore, our factored form is:

$$\boldsymbol{(x-4)(x+1)}$$

Consider an equation where our $a$ term is not 1.

**Example:**

$$6x^2 + 11x + 3$$

Using the box method is effective for this type of problem.

Split the middle term into 2 parts that add to 11 and multiply to 18 ($a * c = 6 * 3 = 18$)

Numbers that could add to 11: $(1 + 10), (2 + 9), (3 + 8), (4 + 7), (5 + 6)$

Factors of 18: $(1 * 18), (2 * 9), (3 * 6)$

The only pair that matches is 2 and 9, so these will be my $b_1$ and $b_2$ terms

| | |
|---|---|
| $6x^2$ | $+2x$ |
| $+9x$ | $+3$ |

Now find the GCF of each row and column

| GCF | $\mathbf{3x}$ | $\mathbf{+1}$ |
|---|---|---|
| $\mathbf{2x}$ | $6x^2$ | $+2x$ |
| $\mathbf{+3}$ | $+9x$ | $+3$ |

Note: when there does not appear to be a GCF, it is 1.

The factored form of $6x^2 + 11x + 3$ is $(3x + 1)(2x + 3)$

**AC Method:**

You may also use the "AC Method" to solve problems that do not have a leading coefficient of 1. The process is very similar to the box method.

First you take $a * c$. In the previous example, this came out to $18$.

Then you find the factors of $18$: $(1 * 18), (2 * 9), (3 * 6)$.

You then take these pairs and see which has a sum of $b$, or $11$ in this case.

$$1 + 18 = 19 \quad 2 + 9 = 11 \quad 3 + 6 = 9$$

Therefore 2 and 9 are the only pair that work. Plug them into the equation for $b$.

$$6x^2 + 2x + 9x + 3$$

Now group them, splitting them right down the middle.

$$(6x^2 + 2x)(9x + 3)$$

Take the GCF of each group(Done properly, the parenthesis should now be the same)

$$2x(3x + 1) + 3(3x + 1)$$

Now factor out the parenthesis-ed term

$$(2x + 3)(3x + 1)$$