

Slope-Intercept Form and Point-Slope Form

Slope of the line	$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form	$y = mx + b$ <i>m is slope; b is y-intercept</i>
Point-Slope Form	$y = m(x - x_1) + y_1$ or $y - y_1 = m(x - x_1)$
Slope of parallel lines	$m_1 = m_2$ (slopes are the same)
Slope of perpendicular lines	$m_1 m_2 = -1$ (slopes are opposite & reciprocal)
Equations of Horizontal and Vertical Lines	$y = b$ horizontal line $x = a$ vertical line, where a & b are constants

Example (1): Write the slope - intercept equation of a line which passes through $(0,-7)$ whose slope is 2.

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope (m) & y-intercept $(0, b)$, and the problem provides both information.

$$m = 2 \text{ and } b = -7 \text{ The equation of the line is } y = 2x - 7$$

Example (2): Write the slope-intercept equation of a line which passes through $(0,4)$ and $(3,-5)$.

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope (m) & y-intercept $(0, b)$, however, we only have y-intercept.

To find the slope,
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 4}{3 - 0} = \frac{-9}{3} = -3$$

$$m = -3 \text{ and } b = 4 \text{ The equation of the line is } y = -3x + 4$$

Example (3): Write the slope-intercept equation of a line which passes through $(-1, 4)$ whose slope is 5.

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope (m) & y-intercept $(0, b)$, however, we only have slope. Here there are two ways to find the equation of the line.

Method I We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b .

$$m = 5, (x_1, y_1) = (-1, 4) \quad 4 = 5(-1) + b$$
$$\Rightarrow b = 9$$

The equation of the line is $y = 5x + 9$

Method II Since we are given slope m and an ordered pair (x_1, y_1) , we can use Point-slope form to find equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$ $y = 5(x - (-1)) + 4$

$$m = 5, (x_1, y_1) = (-1, 4) \quad \Rightarrow y = 5(x + 1) + 4 \quad \text{Simplify the parenthesis}$$

$$\Rightarrow y = 5x + 5 + 4 \quad \text{Distribute 5 into parenthesis}$$

$$\Rightarrow y = 5x + 9$$

Example (4): Write the slope-intercept equation of a line which passes through $(1, 3)$ and $(-5, -1)$.

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope (m) & y-intercept $(0, b)$. However, we are given two ordered pairs (x_1, y_1) and (x_2, y_2) without slope and y-intercept. Therefore, we need to find the slope first. Then we can use the two methods discussed on Example (3) to find the equation of the line.

$$(x_1, y_1) \quad (x_2, y_2)$$

To find the slope between two ordered pairs, $(1,3)$ and $(-5,-1)$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{-5 - 1} = \frac{-4}{-6} = \frac{2}{3}$

Method I Now we have slope, we will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b .

$$m = \frac{2}{3}, (x_1, y_1) = (1, 3) \quad 3 = \frac{2}{3}(1) + b$$

$$\Rightarrow b = \frac{7}{3}$$

The equation of the line is $y = \frac{2}{3}x + \frac{7}{3}$

Method II We also can use Point-slope form to find the equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$ $y = \frac{2}{3}(x - 1) + 3$

$$m = \frac{2}{3}, (x_1, y_1) = (1, 3) \quad \Rightarrow y = \frac{2}{3}x - \frac{2}{3} + 3 \quad \text{Distribute } \frac{2}{3} \text{ into parenthesis}$$

$$\Rightarrow y = \frac{2}{3}x - \frac{2}{3} + \frac{9}{3} \quad \text{Combine like term}$$

$$\Rightarrow y = \frac{2}{3}x + \frac{7}{3}$$

Example (5): Write the slope-intercept equation of a line which is parallel to $y = 4x - 2$, passing through $(1, 3)$.
 x_1, y_1

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are slope (m) & y-intercept ($0, b$). Since the line we're looking for is **parallel** to $y = 4x - 2$, **their slopes are the same**, $m = 4$.

Method I We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b .

$$m = 4, (x_1, y_1) = (1, 3) \quad 3 = 4(1) + b \quad \Rightarrow \quad b = -1$$

The equation of the line is $y = 4x - 1$

Method II We also can use Point-slope form to find the equation of the line.

$$\text{Point-slope form is } y = m(x - x_1) + y_1 \quad y = 4(x - 1) + 3$$

$$m = 4, (x_1, y_1) = (1, 3) \quad \Rightarrow y = 4x - 4 + 3 \quad \text{Distribute 4 into parenthesis}$$

$$\Rightarrow y = 4x - 1 \quad \text{Combine like term}$$

Example (6): Write the slope-intercept equation of a line which is perpendicular to

$$y = -\frac{1}{3}x + 4, \text{ passing through } (x_1, y_1) (-3, 5).$$

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are the slope (m) & y-intercept ($0, b$). Since our line is **perpendicular** to $y = -\frac{1}{3}x + 4$ (which was given), we can find the slope of our line by **taking the opposite sign and using the reciprocal of the given line** which has a slope of $m = -\frac{1}{3}$. Therefore, **the slope of our line is** $m = 3$ (the perpendicular one to the given line)

Method I We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b .

$$m = 3, (x_1, y_1) = (-3, 5) \quad 5 = 3(-3) + b$$

$$\Rightarrow 5 = -9 + b$$

$$\Rightarrow b = 14$$

$$\text{The equation of the line is } y = 3x + 14$$

Method II We also can use Point-slope form to find the equation of the line.

$$\text{Point-slope form is } y = m(x - x_1) + y_1 \quad y = 3(x - (-3)) + 5$$

$$m = 3, (x_1, y_1) = (-3, 5) \quad \Rightarrow y = 3(x + 3) + 5 \quad \text{Simplify the parenthesis}$$

$$\Rightarrow y = 3x + 9 + 5 \quad \text{Distribute 5 into parenthesis}$$

$$\Rightarrow y = 3x + 14$$

Example (7): Write an equation of a vertical line which passes through $(-1,6)$.

Solution:

The equation of a vertical line is $x = a$

The x -coordinate of the point $(-1,6)$ is -1 . Therefore, the equation of the vertical line is $x = -1$

Example (8): Write an equation of a horizontal line which passes through $\left(\frac{3}{4}, -\frac{5}{6}\right)$.

Solution:

The equation of a horizontal line is $y = b$

The y -coordinate of the point $\left(\frac{3}{4}, -\frac{5}{6}\right)$ is $-\frac{5}{6}$. Therefore, the equation of the horizontal line is $y = -\frac{5}{6}$

Exercises:

1. Write the slope - intercept equation of a line which passes through $(0,5)$ whose slope is 4.
2. Write the slope-intercept equation of a line which passes through $(0,-3)$ and $(4,5)$.
3. Write the slope-intercept equation of a line which passes through $(4,0)$ and $(7,-1)$.
4. Write the slope-intercept equation of a line which is parallel to $y = 3x + 5$, passing through $(-6,3)$
5. Write the slope-intercept equation of a line which is perpendicular to $y = 7x + 2$, passing through $(3,2)$
6. Write an equation of a horizontal line which passes through $(5,-1)$
7. Write an equation of a vertical line which passes through $\left(8, \frac{7}{3}\right)$.

Answers:

1. $y = 4x + 5$ 2. $y = 2x - 3$ 3. $y = -\frac{1}{3}x + \frac{4}{3}$ 4. $y = 3x + 21$ 5. $y = -\frac{1}{7}x + \frac{17}{7}$

6. $y = -1$ 7. $x = 8$