Evaluating and Simplifying nth roots

This worksheet will help you to rewrite, simplify and evaluate radical expressions with higher roots (*n*th roots). Radical expressions with higher roots can also be rewritten with rational exponents. Therefore, we are able to simplify and evaluate radical expressions using rational exponents property.

Rational Exponents and Roots

You will need this property $\left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ to rewrite radicals and rational exponents. Assume all variables represent positive real numbers.

Example #1: Rewrite radicals to rational exponents.

a.) $\sqrt{a} = a^{\frac{1}{2}}$ b.) $\sqrt[3]{x^3} = x^{\frac{3}{3}} = x$ c.) $\sqrt[5]{7^2} = 7^{\frac{2}{5}}$ d.) $\sqrt[4]{-2x} = (-2x)^{\frac{1}{4}}$

Example #2: Rewrite rational exponents to radicals.

a.) $y^{\frac{1}{4}} = \sqrt[4]{y}$ b.) $5^{\frac{3}{2}} = \sqrt{5^3}$ c.) $(-27)^{\frac{2}{3}} = \sqrt[3]{(-27)^2}$ or $\left(\sqrt[3]{-27}\right)^2$

Now, we are able to simplify and evaluate the following expressions after knowing how to rewrite radicals and rational exponents. In many problems, you will also need to use integer exponential rules to complete the problems. So, here is a quick review:

$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{m \cdot n}$
$\frac{a^m}{a^n} = a^{m-n}$	$a^{-n} = \frac{1}{a^n}$

$(a^m b^n)^p = a^{mp} b^{np}$	$\left(\frac{a}{b}\right)^{-p} = \left(\frac{b}{a}\right)^{p}$

Example #3: Write each expression in radical notation and then evaluate.

a.) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$ (because $2*2*2*2 = 2^4 = 16$)

b.) $36^{\frac{1}{2}} = \sqrt{36} = 6$ (because $6*6 = 6^2 = 36$)

c.) $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$ (because $4*4*4 = 4^3 = 64$)

d.) $125^{-1/3} = \frac{1}{125^{1/3}} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$ (because $5*5*5 = 5^3 = 125$)

e.) $2^{1/3} * 2^{5/3} = 2^{1/3+5/3} = 2^{6/3} = 2^2 = 4$

f.) $\frac{49^{7/4}}{49^{5/4}} = 49^{7/4-5/4} = 49^{2/4} = 49^{1/2} = \sqrt{49} = 7$

Example #4: Simplify each expression using properties of exponents. Assume that all variables are positive.

a.) $(x^2)^{\frac{3}{2}} = x^{\frac{2}{1}*\frac{3}{2}} = x^3$

b.) $\left(y^{\frac{1}{2}}\right)^{-\frac{4}{3}} = y^{\frac{1*-4}{2*3}} = y^{-\frac{2}{3}} = \frac{1}{y^{2/3}} = \frac{1}{\sqrt[3]{y^2}}$

$y \; x^6 \; x^{1\frac{6*2}{3}} \quad 4 \quad x^4$

$x^4 x^{\frac{2}{3}}$

c.) $(27)^{\frac{2}{3}} = 27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

d.) $\left(\frac{x^6}{8}\right)^{-\frac{1}{3}} = \left(\frac{8}{x^6}\right)^{\frac{1}{3}} = \frac{8^{\frac{1}{3}}}{x^{6*\frac{1}{3}}} = \frac{\sqrt[3]{8}}{x^2} = \frac{2}{x^2}$

e.) $\left(81y^4 z^8\right)^{\frac{1}{4}} = 81^{\frac{1}{4}} y^{4*\frac{1}{4}} z^{8*\frac{1}{4}} = 3yz^2 \quad (81^{\frac{1}{4}} = \sqrt[4]{81} = 3)$

f.) $\sqrt[3]{125x^9} = \left(125x^9\right)^{\frac{1}{3}} = 125^{\frac{1}{3}} x^{9*\frac{1}{3}} = 5x^3$

Exercises:

Write each expression in radical notation and then evaluate.

1. $125^{\frac{2}{3}}$ **2.** $27^{-\frac{1}{3}}$ **3.** $(-32)^{\frac{3}{5}}$

$\frac{1}{3}$ **3.** -8 **4.** $\frac{x^2}{16}$ **5.** $6y z^{2\,3}$ **6.** $2x y^{2\,4}$

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Simplify each expression using properties of exponents. Assume that all variables are positive.

4. $\left(64x^{3}\right)^{\frac{2}{3}}$ **5.** $\left(36y^{4}z^{6}\right)^{\frac{1}{2}}$ **6.** $\sqrt[3]{8x^{6}y^{12}}$

Answers:

1. 25 **2.**