

Monomial Factors

Factoring means starting with a product and determining what was multiplied together to get that product.

When factoring polynomials, the first step is to check for any **common factors**.

Numerical Examples

The number **24** can be factored in several ways:

- $24 = 1 \times 24$
- $24 = 2 \times 12$
- $24 = 3 \times 8$
- $24 = 4 \times 6$

Although there are several answers, breaking each number into **prime factors** shows there is really only one set:

- $24 = 2 \times 2 \times 2 \times 3$

So, the prime factorization of 24 is $2 \times 2 \times 2 \times 3$.

Greatest Common Factor (GCF)

- For integers, the GCF is the **largest number** that divides evenly into all given integers.
 - For variables, the GCF is the **highest power of each variable** that appears in all terms.
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Example A: Integers

Find the GCF of **36** and **42**

- $36 = 2 \times 2 \times 3 \times 3$
- $42 = 2 \times 3 \times 7$

GCF = 6

Example B: Integers

Find the GCF of **18, 48, 54**

- $18 = 2 \times 3 \times 3$
- $48 = 2 \times 2 \times 2 \times 2 \times 3$

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$$- 54 = 2 \times 3 \times 3 \times 3$$

$$\text{GCF} = 6$$

Example C: Variables

Find the GCF of x^2y^3 and x^3y^4

$$- x^2y^3 = x \cdot x \cdot y \cdot y \cdot y$$

$$- x^3y^4 = x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$$

$$\text{GCF} = x^2y^3$$

Example D: Variables

Find the GCF of a^4bc^3 and a^2b^2

$$- a^4bc^3 = a \cdot a \cdot a \cdot a \cdot b \cdot c \cdot c \cdot c$$

$$- a^2b^2 = a \cdot a \cdot b \cdot b$$

$$\text{GCF} = a^2b$$

Example E: Variables with Coefficients

Find the GCF of $24x^2y^4$ and $18xy^3$

$$- 24x^2y^4 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$$

$$- 18xy^3 = 2 \cdot 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$$

$$\text{GCF} = 6xy^3$$

Factoring Using the GCF

Example 1

Factor $3x^4 - 9x^2$

1. GCF of coefficients = 3

2. GCF of variables = x^2

Factor out GCF: $3x^2(x^2 - 3)$

Example 2

Factor $6a^3b^2 - 12ab^2 + 6b$

1. GCF of coefficients = 6

2. GCF of variables = b

Factor out GCF: $6b(a^2b^2 - 2a + 1)$

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Example 3

Factor $8x^3y^5 - 4x^2y^3 - xy^2$

1. GCF of coefficients = 1

2. GCF of variables = xy^2

Factor out GCF: $xy^2(8x^2y^3 - 4x + -1)$

Practice Problems

Factor each expression:

a) $16x^4 - 8x^7$

b) $8y + 12$

c) $14x^2 - x$

d) $y^3 + 4y^2 + 3y$

e) $3a^2b^2 - 9ab^2 + 15b^2$

f) $3a^2 - 2b^3 + 7$

g) $14x^3y^3 + 28x^2y^2 - 3xy$

h) $20a^3b^2 - 15ab^3 + 5ab^2$

i) $x^4y^4 - 3x^3y^3 + 6x^2y^2$

j) $16x^3y^2 - 8x^2y^3 - 48xy^2$

Answer Key

a) $8x^4(2 - x^3)$

b) $4(2y + 3)$

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c) $x(14x - 1)$

d) $y(y^2 + 4y + 3)$

e) $3b^2(a^2 - 3a + 5)$

f) No common factors (GCF = 1)

g) $xy(14x^2y^2 + 28x - 3)$

h) $5ab^2(4a^2 - 3b + 1)$

i) $x^2y^2(x^2y^2 - 3xy + 6)$

j) $8xy^2(2x^2 - xy - 6)$