

Multiplying and Dividing Decimal Numbers

Important Ideas in Multiplication:

1. When multiplying decimals it is not necessary to line up the decimal points.
2. The decimal point is inserted in the product after the multiplication has been done.
3. It is sometimes necessary to insert zeros as place holders in the product.

To multiply decimal numbers

1. Arrange the numbers vertically as if they were whole numbers. (As multiplication is commutative the order in which the numbers are arranged does not matter).
2. Do **not** line up the decimal points.
3. Multiply the digits as if multiplying whole numbers. Remember to start each partial product underneath the digit you are multiplying by.
4. Count up the total number of decimal places in the two numbers you are multiplying.
5. Count over the same number of places in the product going from right to left and then insert the decimal point. You may have to insert zeros in order to obtain the correct number of decimal places.

We will now work through three examples.

Example 1: Multiply: 3.65×8.2

Arrange the numbers as if they were whole numbers. Do **not** line up the decimal points.

$$\begin{array}{r} 3.65 \\ \times \quad \underline{8.2} \\ \hline \end{array}$$

Multiply as if multiplying whole numbers.

Start each partial product underneath the digit you are multiplying by.

$$\begin{array}{r} 3.65 \\ \times \quad \underline{8.2} \\ \hline \end{array}$$

730

2920

29930

Now count the number of decimal places in the two numbers you are multiplying and insert the decimal point so that the number of decimal places in the product is the same as the total number of decimal places in the two numbers being multiplied.

$$\begin{array}{r} 3.65 \text{ 2 decimal places} \quad \longleftarrow \\ \times \quad \underline{8.2} \text{ 1 decimal place} \quad \longleftarrow \\ \hline \quad \quad \quad 730 \\ \quad \quad \underline{2920} \\ \quad 29930 \quad \longleftarrow \text{ 3 decimal places} \end{array}$$

As the last digit in the decimal part of the product is a zero, it can be omitted without changing the value of the number. The final product is 29.93.

Example 2: Multiply: 0.034×0.37

Arrange the numbers as if they were whole numbers. Do **not** line up the decimal points.

$$\begin{array}{r} 0.034 \\ \times \quad \underline{0.37} \\ \hline \end{array}$$

Multiply as if multiplying whole numbers.

Start each partial product underneath the digit you are multiplying by.

$$\begin{array}{r} 0.034 \\ \times \quad \underline{0.37} \\ \hline \quad \quad \quad 238 \end{array}$$

$$\begin{array}{r} \underline{102} \\ 1258 \end{array}$$

Now count the number of decimal places in the two numbers you are multiplying and insert the decimal point so that the number of decimal places in the product is the same as the total number of decimal places in the two numbers being multiplied.

$$\begin{array}{r} 0.0343 \text{ decimal places} \\ \times \quad \underline{0.37} \text{ 2 decimal places} \\ \hline 238 \\ \underline{102} \\ 0.01258 \end{array} \quad \begin{array}{l} \longleftarrow \\ \longleftarrow \\ \longleftarrow \text{ 5 decimal places} \end{array}$$

Note that we had to insert a zero in the product after the decimal point in order to have enough decimal places.

Example 3: Multiply: 0.758×24

Arrange the numbers as if they were whole numbers. Do **not** line up the decimal points.

$$\begin{array}{r} 0.758 \\ \times \quad \underline{24} \end{array}$$

Multiply as if multiplying whole numbers.

Start each partial product underneath the Digit you are multiplying by.

$$\begin{array}{r} 0.758 \\ \times \quad \underline{24} \end{array}$$

$$\begin{array}{r} 3032 \\ \underline{1516} \\ 18192 \end{array}$$

Now count the number of decimal places in the two numbers you are multiplying and insert the decimal point so that the number of decimal places in the product is the same as the total number of decimal places in the two numbers being multiplied.

$$\begin{array}{r} 0.7583 \text{ decimal places} \\ \times \quad \underline{24.0} \text{ decimal places} \\ \hline 3032 \\ \underline{1516} \\ 18.192 \end{array} \quad \begin{array}{l} \longleftarrow \\ \longleftarrow \\ \longleftarrow \text{ 3 decimal places} \end{array}$$

Note that because 24 is a whole number it has zero decimal places.

Important Ideas in Division

1. In order to divide decimals without a calculator, the divisor (the number you are dividing **by**) must be a whole number.
2. To make the divisor into a whole number, the decimal point is moved to right as many places as necessary.
3. Whenever the decimal point in the divisor is moved, the decimal point in the dividend (the number you are dividing) must also be moved **the same number of places**.
4. Moving the decimal points one place to the right is the same as multiplying both numbers by 10. Moving the decimal points two places to the right is the same as multiplying both numbers by 100. Moving the decimal points three places to the right is the same as multiplying both numbers by 1,000 etc.
5. If the divisor is already a whole number you **do not move** the decimal points in either the divisor or the dividend even if the dividend is a decimal number.
6. The decimal points in the quotient (answer) and the dividend must be kept lined up.
7. It is often necessary to insert zeros as place holders in the dividend.
8. It is often necessary to round the quotient to a given place value.

To Divide Decimal Numbers

1. Rewrite the division in a horizontal format with the divisor on the outside of the division symbol and the dividend inside the division symbol.
2. Move the decimal point in the divisor (the number you are dividing **by**) to the right as many places as necessary to make it a whole number.
3. Move the decimal point in the dividend (the number you are dividing) the same number of places as you moved the decimal point in the divisor.
4. Divide as if dividing whole numbers being sure to keep the decimal points lined up.

5. If the division does not terminate, add zeros as place holders and continue dividing to one decimal place beyond the place you are rounding to.

We will now work through four examples.

Example 4: Divide: $2.406 \div 0.6$

Rewrite the division.

$$0.6 \overline{)2.406}$$

Move the decimal point one place to the right in both the divisor and the dividend.

$$6 \overline{)24.06}$$

Divide as if dividing whole numbers. Keep the decimal points lined up.

$$\begin{array}{r} 4.01 \\ 6 \overline{)24.06} \\ \underline{24} \\ 00 \\ \underline{0} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

The remainder is zero so this is a terminating decimal.

Note that every time you bring a number down you must try to divide. If you cannot divide (the divisor is greater than the number you are dividing), a zero must be recorded in the quotient.

Example 5: Divide: $0.0275 \div 0.16$ Round to the nearest thousandth.

Rewrite the division.

$$0.16 \overline{)0.0275}$$

Move the decimal point two places to the right $\overline{)16} 2.75$ in both the divisor and the dividend. 0.1718 or 0.172

Divide as if dividing whole numbers.

Keep the decimal points lined up.

$$\begin{array}{r}
 \overline{)16} 2.7500 \\
 - \underline{16} \\
 115 \\
 - \underline{112} \\
 30 \\
 - \underline{16} \\
 140 \\
 - \underline{128} \\
 120 \\
 - \underline{120} \\
 0000
 \end{array}$$

Note that we have divided to the ten-thousandths place in order to round to the thousandths place. Recall that if the digit immediately to the right of the place we are rounding to is a 4 or less we round down, and if it is a 5 or more we round up.

As the digit to the right of the thousandths place is an “8” the final quotient is 0.172 rounded to the nearest thousandth.

Example 6: Divide: $45.52 \div 47$ Round to the nearest hundredth.

Rewrite the division.

$$\overline{)47} 45.52$$

The divisor is a whole number so do not move the decimal points.

Divide as if dividing whole numbers.

Keep the decimal points lined up.

$$\begin{array}{r}
 0.968 \quad \text{or } 0.97 \\
 \overline{)47} 45.520 \\
 - \underline{423} \\
 322 \\
 - \underline{282} \\
 400 \\
 - \underline{400} \\
 0000
 \end{array}$$

$$- \underline{376}$$

Note that we have divided to the thousandths place in order to round to the hundredths place. Recall that if the digit immediately to the right of the place we are rounding to is a 4 or less we round down, and if it is a 5 or more we round up.

As the digit to the right of the thousandths place is an “8” the final quotient is 0.97 rounded to the nearest hundredth.

Example 7: Divide: $672.9 \div 0.32$ (Round to the nearest tenth)

Rewrite the division.

$$\begin{array}{r} 0.32 \overline{)672.9} \\ 32 \overline{)672900} \end{array}$$

Move the decimal point two places to the right in both the divisor and the dividend.

Note that we had to insert a zero after the “9”

$$\begin{array}{r} 2102.81 \\ 32 \overline{)67290.00} \end{array} \text{ Divide}$$

as if dividing whole numbers.

Keep the decimal points lined up.

$$- \underline{64}$$

$$\begin{array}{r} 32 \\ - \underline{32} \\ 09 \\ - \underline{00} \\ 90 \end{array}$$

$$- \underline{64}$$

$$260$$

$$- \underline{256}$$

$$40$$

$$- \underline{32}$$

Note that we have divided to the hundredths place in order to round to the tenths place.

As the digit to the right of the tenths place is a “1” we round down and the final quotient is 2102.8 rounded to the nearest tenth.

Practice Problems

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|----|-------------------------------------|-----|---|
| 1. | 23.7×1.2 | 2. | 0.037×0.09 |
| 3. | 89.62×8 | 4. | 2.187×0.45 |
| 5. | $2.436 \div 0.6$ | 6. | $0.1314 \div 0.018$ |
| 7. | $4.817 \div 16$ (nearest hundredth) | 8. | $0.1142 \div 17.2$ (nearest thousandth) |
| 9. | $18.4 \div 7.3$ (nearest tenth) | 10. | $27.738 \div 60.3$ (nearest hundredth) |

Answers to Practice Exercises

- 28.44
- 0.00333
- 716.96
- 0.98415
- 4.06
- 7.3
- 0.30
- 0.007
- 2.5
- 0.46