Simplifying Rational Expressions

A rational expression is a quotient of two monomials and/or polynomials.

Examples: 2, $5x^2 + 2x$, $5y^2$, $2x+1$, etc.

Beginning Steps:

1. Factor out the greatest common factor (GCF), if any, in all numerators and denominators.
2. Factor completely all numerators and denominators.
3. Remaining steps depend on the type of problem. See examples below for three types of problems: (I) single expression; (II) products and quotients; (III) sums and differences.

Type I: Single Rational Expression

Example #1:

Simplify: $(3x + 6) / 3x$

Solution:

1. Factor out GCF (of 3) in the numerator: $3(x + 2)$

2. There's no other factorization to be done.

3. Cancel like factors (representing 1).

Note: It is incorrect to cancel as follows: $(3x + 6) / 3x \neq x + 2$

Example #2:

Simplify: $(150x - 6x^3) / (6x^2 - 27x - 15)$

Solution:

1. Factor out the GCF of 6x in the numerator and of 3 in the denominator.

2. Factor the numerator and denominator completely.

3. Cancel like factors (representing 1).

Note: $5 - x = -1(x - 5)$

Type II: Products and Quotients

Example #3:

Simplify: $(x^2 + 1) \div (x^2 + 8x + 7) \bullet (2x - 3) / (4x^2 - 9)$

Solution:

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1. Change division to multiplication by the reciprocal.

2. Factor completely and cancel.

3. Write final answer as one fraction.

Example #4:

Simplify: $(2x^2 - 9x - 5) \bullet (6x - 2) / (3x - 1)(x^2 + 2x - 8)$

Solution:

1. Factor completely.

2. Cancel like factors.

Type III: Sums and Differences

Example #5:

Add and simplify: $(y^2 + 12y + 20)/(y + 10)(y + 2) + (3y + 7)/(y^2 - 100)$

Solution:

1. Factor denominators and find LCD.

2. LCD is $(y + 10)(y + 2)(y - 10)$.

3. Write all fractions with LCD.

4. Add fractions.

5. If possible, factor and reduce.

Example #7:

Simplify: $(2x^2 + 13x + 6)/(x + 3) - (6x + 3)/5$

Solution:

1. Factor all denominators and find LCD.

2. LCD is $3(2x + 1)(x + 6)$.

3. Write all fractions with LCD.

4. Subtract fractions.

5. Cannot factor or reduce further.