

Solving Linear Inequalities and Compound Inequalities

Steps for solving linear inequalities are similar to those for solving linear equations. The main differences are:

- Multiplying or dividing by a negative constant reverses the inequality sign.
- Expressing the solution set using interval notation or set-builder notation.

Solve Linear Inequalities

Example (1): $3x + 8 > 6x$

- Subtract 8 on each side: $3 > 6x - 8$
- Divide by 3: $x > -2/3$
- Solution set: $\{x \mid x > -2/3\}$

Example (2): $3x - 2 \geq 5x + 13$

- Add 2 to each side: $3x \geq 5x + 15$
- Subtract $5x$: $-2x \geq 15$
- Divide by -2 (reverse inequality): $x \leq -15/2$
- Solution set: $\{x \mid x \leq -15/2\}$

Example (3): $6(3+4x) - 2 < 20$

- Expand: $18 + 24x - 2 < 20$
- Simplify: $24x + 16 < 20$
- Subtract 16: $24x < 4$
- Divide by 24: $x < 1/6$
- Solution set: $\{x \mid x < 1/6\}$

Example (4): $(w-3)-2(2-w) \leq 2$

- Expand: $w - 3 - 4 + 2w \leq 2$
- Simplify: $3w - 7 \leq 2$
- Add 7: $3w \leq 9$
- Divide by 3: $w \leq 3$
- Solution set: $\{w \mid w \leq 3\}$

Example (5): $(5z-4)/3 > (2+5z)/5$

- Find LCD=15, multiply through: $3(5z-4) > 5(2+5z)$
- Expand: $15z - 12 > 10 + 25z$
- Simplify: $-10z > 22$
- Divide by -10 (reverse sign): $z < -11/5$
- Solution set: $\{z \mid z < -11/5\}$

Solving Linear Inequalities and Compound Inequalities

Interval Notation

- Use () if the endpoint is not included (strict inequalities $<$ or $>$).
- Use [] if the endpoint is included ($\leq$ or $\geq$).
- Use $\cup$ for union when combining two intervals.

Solve Compound Inequalities

Example (1): $x < 3$ and $x \geq -4$

- Interval: $[-4, 3)$

Example (2): $x < 3$ or $x \geq -4$

- Interval: $(-\infty, \infty)$

Example (3): $x+1 < 9$ and $2x-1 > 7$

- Interval: $(4, 8)$

Example (4): $x+1 < 2$ or $2x-1 > 8$

- Interval: $(-\infty, 1) \cup (4, \infty)$

Example (5): $-5 < x+3 < 9$

- Interval: $(-8, 6)$

Example (6): $-2 < 7-3x \leq 19$

- Interval: $[-4, 3)$

Exercises

1. $2x+1 \leq -1$ or $2x+1 \geq 3$
2. $-1 < 5-2x \leq 11$
3. $2t-3 \geq 5t-(2t+1)$
4. $3x-2 < 2x+1$

Answers

1. $(-\infty, -1] \cup [1, \infty)$
2. $[-3, 3)$
3. $(-\infty, -2]$
4. $(-\infty, 3)$